if the British banks would agree to issue statistics that would bear comparison with those now published by the member banks of the Federal Reserve System. It is confessedly a forlorn hope. Hence a proper understanding of the sparse information now available is especially essential to American and other foreign investigators.

THE PROBABLE ERROR OF THE DIFFERENCE BETWEEN TWO DIFFERENCES

By J. Arthur Harris

Given measures of two pairs of variables, w, x, and y, z, it is sometimes necessary to determine the probable error of the difference, d_1-d_2, where $d_1=x-w$, $d_2=z-y$.

A formula has doubtless been given somewhere in the extensive literature, but I do not recall having seen it. Since it must involve the value of $r_{d_1d_2}$ when there is any reason to suspect that the two differences may be correlated, a highly convenient method (which may be used when the correlations between the variables themselves are required and when variation and correlation are such that the ordinary theory of probable errors may be employed) is that based on the moments and product moments of the original variates, taken about 0 as origin. The formula for $r_{d_1d_2}$ has already been suggested [1] and employed in the study of growth increments.[2]

The moments $\Sigma(w)$, $\Sigma(w^2)$, $\Sigma(x)$, $\Sigma(x^2)$, . . ., and the product moments $\Sigma(wx)$, $\Sigma(wy)$, $\Sigma(wz)$, $\Sigma(xy)$, . . ., are generally required for other purposes of the investigation.

The constants for the differences are given by well-known formulas:

$$\bar{d}_1=\bar{x}-\bar{w},$$

$$\sigma_{d_1}{}^2=[\Sigma(w^2)+\Sigma(x^2)-2\Sigma(wx)]/N-\bar{d}_1{}^2,$$

and similarly for d_2, where the bars denote means and the sigmas denote standard deviations.

The product moment from which $r_{d_1d_2}$ may be deduced is given by

$$\Sigma(d_1d_2)=\Sigma(wy)+\Sigma(xz)-\Sigma(xy)-\Sigma(wz).$$

The standard deviation of d_1-d_2 may then be written

$$\sigma_{(d_1-d_2)}{}^2=\sigma_{d_1}{}^2+\sigma_{d_2}{}^2-2r_{d_1d_2}\sigma_{d_1}\sigma_{d_2}.$$

[1] Harris, J. Arthur, "Formulae for the determination of the correlation of size and of growth increments in the developing organisms." *Proc. Soc. Exp. Biol. Med.*, Vol. XVIII, pp. 4–5, 1921.

[2] Harris, J. Arthur, and Reed, H. S. "Inter-periodic correlation in the analysis of growth." *Biol. Bull.*, Vol. XL, pp. 243–258, 1921.